

## Sorting modes

### Waves in (and on) plates

Mechanical waves on plates have been extensively investigated in recent years. One important application is the detection of cracks or erosions in structures. Simulations have been used to understand the various types of mechanical waves and the effect of defects, surface layers, etc. on wave propagation. Another important concern is the generation and detection of these waves—often using piezoelectric transducers.

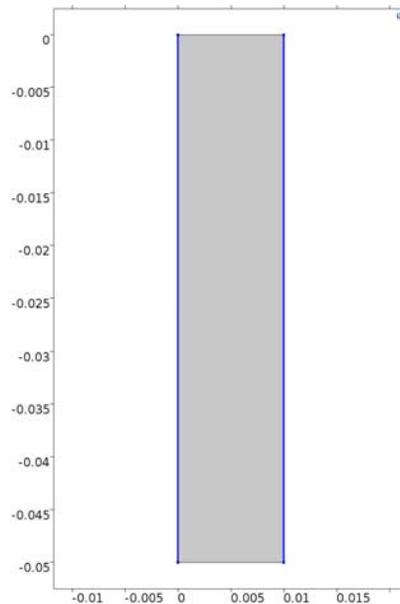
There are several types of waves that have greatly different properties. Lamb waves have displacements throughout the plate thickness. Lamb waves can be symmetric or asymmetric and there will be many modes of each type if the plate is thick. (An analogy can be made to radio frequency waveguides: there are few modes when the dimensions are of the order of a wavelength, and many modes when dimensions become larger). Rayleigh or surface waves have significant displacements close to a surface. Stoneley waves only have significant displacements near a boundary between two materials. With any of these wave types, a starting point of analysis is the use of eigenmode simulations to find the frequencies corresponding to waves of a particular wavelength.

The large number of modes leads to complications. Suppose we are interested in Rayleigh waves and their interaction with surface layers. We use a domain that is thick enough that the bottom surface has minimal influence on the surface waves. Now there are a great many modes close in frequency to the surface waves of interest. How do we "find" these waves?

One way to do this is to manually search through the modes found by an eigenmode calculation until the desired mode is found. This can be tedious, and here we suggest an alternate approach.

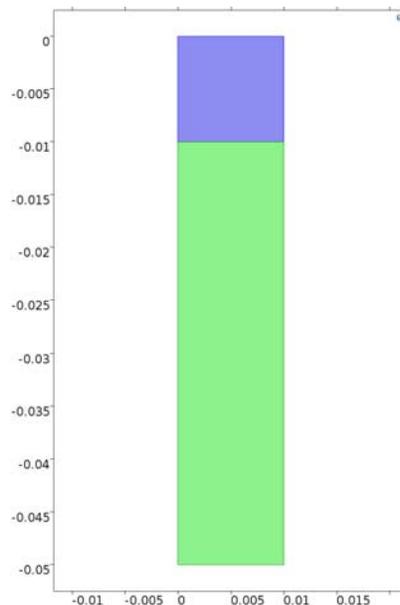
### An example- Rayleigh waves on an aluminum plate

We seek to find the frequency of a Rayleigh wave with a wavelength of 1 cm. We therefore construct a 2D domain 1 cm in width and 5 cm in height. We impose periodic boundary conditions on the right and left edges, and perform an eigenmode calculation. The 5 cm thickness is chosen to be several wavelengths so that the boundary condition on the bottom surface does not significantly influence waves on the top surface. The geometry is shown below.

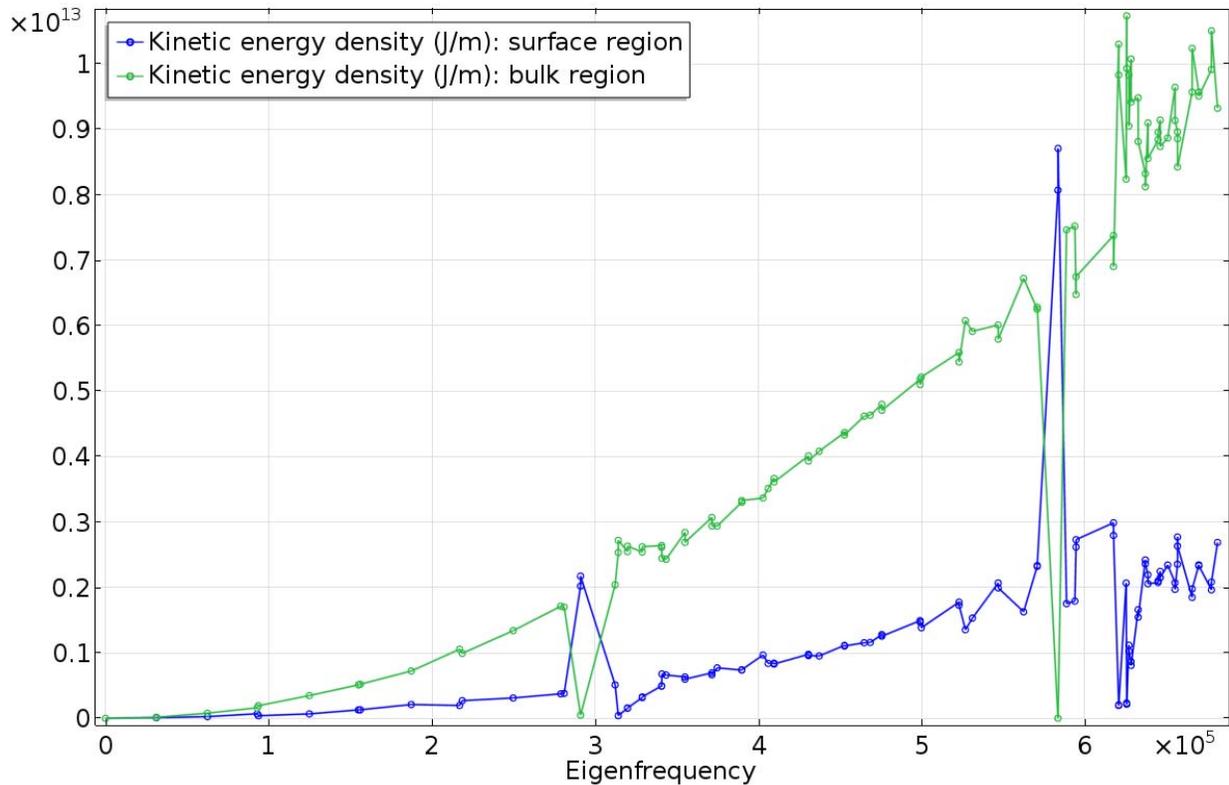


The transverse wave velocity in aluminum is about 3150 m/sec, so we expect Rayleigh waves at a frequency somewhat less than 315 kHz. There are about 10 different modes that *could* be Rayleigh waves. Do we need to look at all of them?

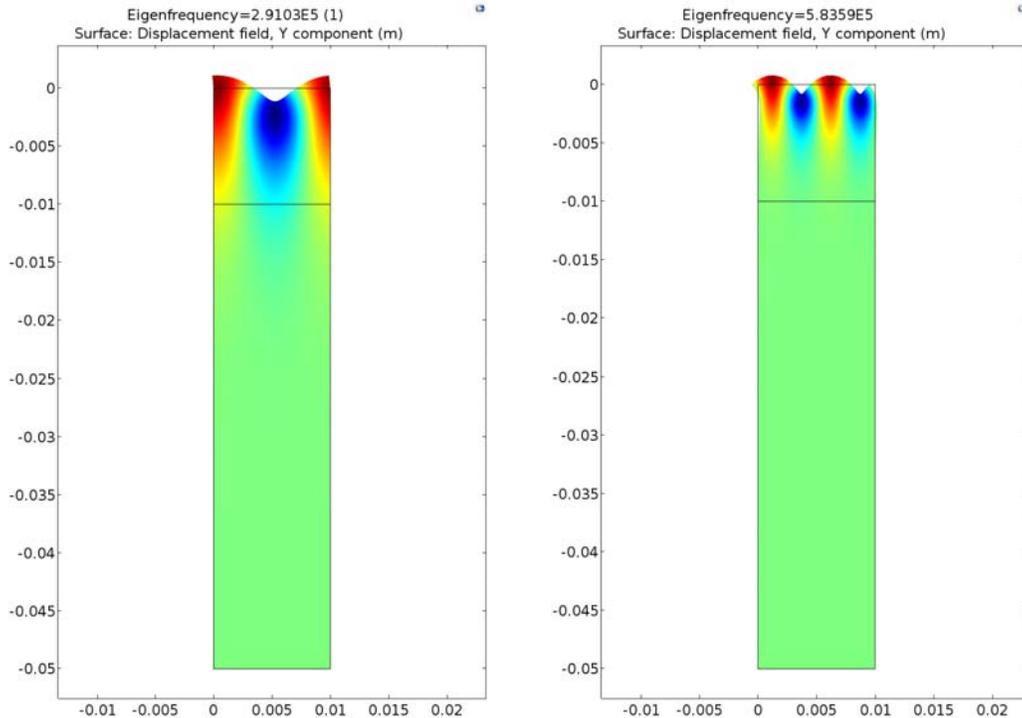
One way to avoid this is to come up with a scheme to classify the modes into Rayleigh-like and not Rayleigh-like. Rayleigh waves have displacements (and thus strain and kinetic energy) close to the surface. Let's divide the simulation domain into a surface and bulk region, and calculate the energy in each region. The figure below shows the surface and bulk domains. Both are aluminum but now we can evaluate the strain or kinetic energy in both of them.



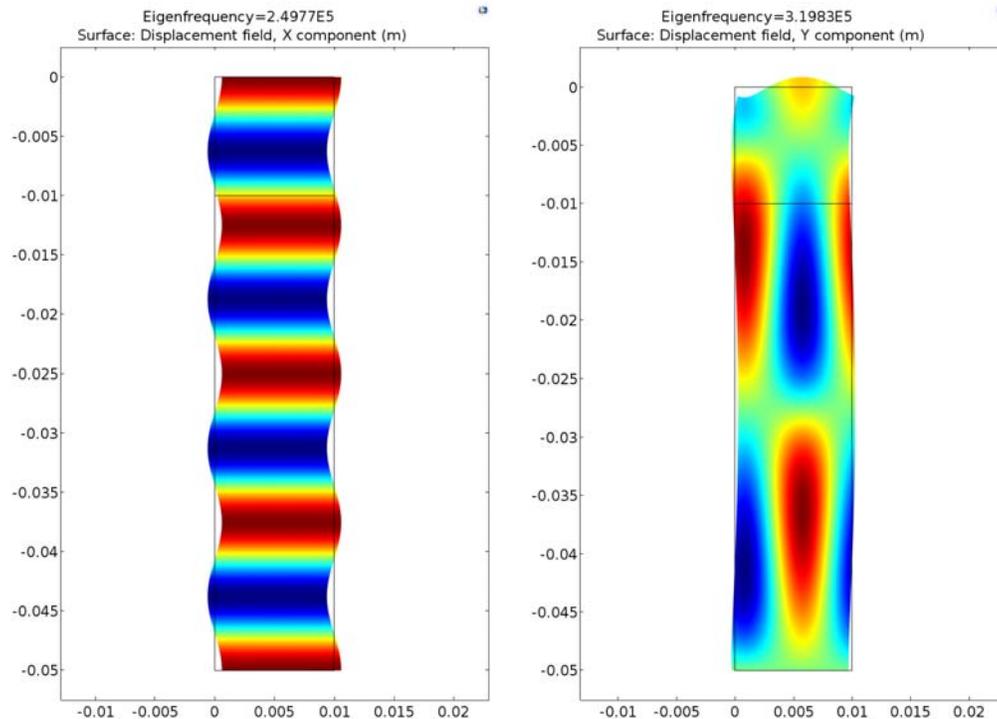
We use surface integration of the quantity `solid.Wk` over each domain and plot the result as a function of eigenfrequency. The figure below shows the result for the first 100 eigenmodes.



Notice that there are only two frequencies that correspond to high kinetic energy density near the surface and low kinetic energy in the bulk. Those two eigenmodes are shown below; the higher frequency mode corresponds to two wavelengths/cm. In these plots color is proportional to the y displacement and the deformed domain is shown. These modes have a velocity of 2910 m/sec- as expected for a Rayleigh wave, a bit less than the transverse wave velocity.



Most of the other modes are various plate wave modes. All are symmetric modes because of the boundary condition imposed on the bottom surface. Below are shown two modes close to the Rayleigh wave mode. The left plot shows an SH plate wave mode (in this plot color represents the x displacement). The right shows a symmetric plate mode.

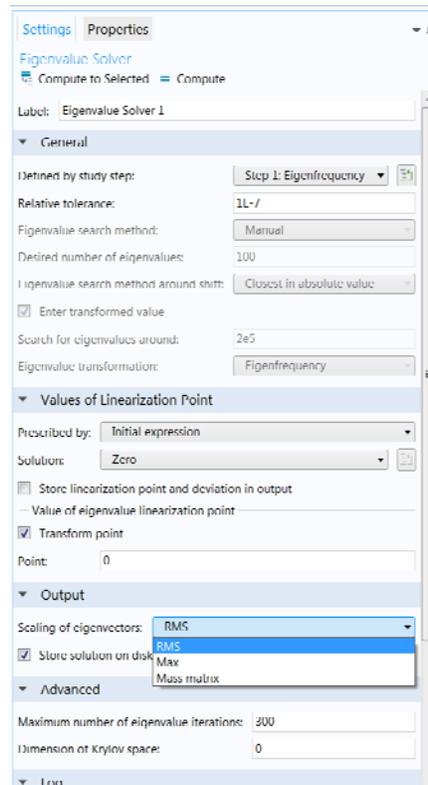


### Normalization of eigenmodes

Ignoring the Rayleigh wave modes, the plot of integrated kinetic energy shows a nearly monotonic increase with frequency. What are we to make of that?

To understand this behavior, we need to understand the meaning of the magnitude of displacements in the eigenmodes. When we perform an eigenfrequency calculation, for each mode we get the eigenfrequency and a map of the displacements in the domain that correspond to that eigenfrequency. *The scaling of that map is arbitrary.*

Comsol offers three options for the scaling of the eigenmodes, found in the properties box for the eigenvalue solver as shown below. We consider here the options **RMS** and **Max**.



The **RMS** option scales the eigenmodes so that the average of the RMS displacement over the domain is roughly<sup>1</sup> constant. This is the default option. The other option is to scale the eigenmodes so that the maximum value of the displacement is constant.

The general upward trend for the surface integral occurs because the kinetic energy density is proportional to the square of the velocity. Velocity is proportional to frequency multiplied by displacement, so the energy increases according to frequency squared.

### Sorting other modes

Here we have considered the specific case of finding surface wave modes. This general idea can be adapted to other types of modes. For example, different expressions can be plotted to highlight Stoneley modes, symmetric or asymmetric modes, etc.

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<sup>1</sup> The RMS scaling is within  $\pm 10\%$  except for Rayleigh modes, which are scaled 50% too low. The Max scaling option is more precise.